Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2022

Mathematics: applications and interpretation

Higher level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) (i) $(m =)\ 54(\%)$ ***A1***

(ii) $(n =)\ 14(\%)$ ***A1***

(iii) $(p =)\ 22(\%)$ ***A1***

(iv) $(q =)\ 10(\%)$ ***A1***

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[4 marks]

(b) (i) $0.54\ \left(\frac{54}{100}, \frac{27}{50}, 54\%\right)$ ***A1***

(ii) $\frac{54}{64}\ \left(0.844, \frac{27}{32}, 84.4\%, 0.84375\right)$ ***A1A1***

Note: Award ***A1*** for a correct denominator (0.64 or 64 seen), ***A1*** for the correct final answer.

[3 marks]

(c) (i) recognizing Binomial distribution with correct parameters ***(M1)***

$X \sim \mathrm{B}(10, 0.68)$

$\left(\mathrm{P}(X = 5) =\right)\ 0.123\ \ (0.122940..., 12.3\%)$ ***A1***

(ii) $1 - \mathrm{P}(X \leq 3)$ **OR** $\mathrm{P}(X \geq 4)$ **OR** $\mathrm{P}(4 \leq X \leq 10)$ ***(M1)***

$0.984\ (0.984497..., 98.4\%)$ ***A1***

(iii) $(0.68)^9 \times 0.32$ ***(M1)***

recognition of two possible cases ***(M1)***

$2 \times \left((0.68)^9 \times 0.32\right)$

$0.0199\ \ (0.0198957..., 1.99\%)$ ***A1***

[7 marks]

(d) **EITHER**

the probability is not constant ***A1***

OR

the events are not independent ***A1***

OR

the events should be modelled by the hypergeometric distribution instead ***A1***

[1 mark]

Total [15 marks]

2. (a) (i) B ***A1***

(ii) F ***A1***

[2 marks]

(b) correct substitution into the midpoint formula ***(M1)***

$\frac{8+5}{2}$

$y = 6.5$ ***A1***

Note: Answer must be an equation for the ***A1*** to be awarded.

[2 marks]

(c) midpoint $= (5, 7)$ ***(A1)***

correct use of gradient formula ***(M1)***

$\frac{8-6}{7-3}$

gradient of $\text{BC} = 0.5$ ***(A1)***

negative reciprocal of gradient ***(M1)***

perpendicular gradient $= -2$

$y - 7 = -2(x - 5)$ (or $y = -2x + 17$) ***A1***

Note: Do not follow through within the part for the final ***A1***.

[5 marks]

(d) (i) attempt to find the intersection of two perpendicular bisectors (BC & CD) ***(M1)***

Note: This may be seen graphically or algebraically.

$6.5 - 7 = -2(x - 5)$ **OR** $6.5 = -2x + 17$

Note: Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x - 5)$ **OR** $y = 4x - 14.5$

$x = 5.25, y = 6.5$ **OR** $(5.25, 6.5)$ ***A1***

Note: The x-coordinate must be exact or expressed to at least 3 sf.

(ii) their correct substitution into distance formula ***(M1)***

$\sqrt{(5.25-7)^2 + (6.5-5)^2}$

$= 2.30 \text{ km} \left(2.30488..., \frac{\sqrt{85}}{4} \right)$ ***A1***

[4 marks]

Total [13 marks]

3. (a) (i) $f'(x) = \frac{-2x}{50} + 2 \quad \left(= \frac{-x}{25} + 2, -0.04x + 2 \right)$ ***A1A1***

Note: Award ***A1*** for each correct term. Award at most ***A0A1*** if extra terms are seen.

(ii) $0 = \frac{-x}{25} + 2$ **OR** sketch of $f'(x)$ with x-intercept indicated ***M1***

$x = 50$ ***A1***

$y = 80$ ***A1***

$(50, 80)$

Note: Award ***M0A0A1*** for the coordinate $(50, 80)$ seen either with no working or found from a graph of $f(x)$.

[5 marks]

(b) (i) $\int_0^{70} \frac{-x^2}{50} + 2x + 30 \, \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location. Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.

(ii) (Area =) $4710 \text{ m}^2 \quad \left(4713.33\ldots, \frac{14140}{3} \right)$ ***A2***

[4 marks]

(c) (i) $\frac{11.4}{4713.33...} \times 100\%$ **OR** $\left| \frac{4701.93... - 4713.33...}{4713.33...} \right| \times 100\%$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the percentage error formula.

$0.242\% \quad (0.241867...\%)$ ***A1***

Note: Percentage sign is required. Accept $0.242038...\%$ if 4710 is used.

(ii) **EITHER**

reduce the width of the intervals (trapezoids) ***A1***

OR

increase the number of intervals (trapezoids) ***A1***

Note: Accept equivalent statements. Award ***A0*** for the ambiguous answer "increase the intervals".

[3 marks]

continued…

Question 3 continued

(d) (i) width of the square is $70-x$ **OR** the length of the square is $\frac{-x^2}{50}+2x+30$

(M1)

Note: Award ***(M1)*** for $70-x$ seen anywhere. Accept $\frac{-x^2}{50}+2x+30$ but only if this expression is explicitly identified as a dimension of the square.

in term of x, equating the length to the width ED ***(M1)***

$\frac{-x^2}{50}+2x+30=70-x$

$(x=14.7920...$ or $135.21)$

$(x=)\ 14.8\ \mathrm{m}\ (14.7920...)$ ***A1***

Note: Award ***M0M0A0*** for an unsupported answer of 15. Award at most ***M1M0A0*** for an approach which leads to $A'(x)=0$. This will lead to a square base which extends beyond the east boundary of the property. Similar for any solution where F is not on the northern boundary, or GH is not on the east boundary.

(ii) **EITHER**

$(70-14.7920...)^2$ ***(M1)***

OR

$(55.2079...)^2$ ***(M1)***

OR

$\left(\frac{-(14.7920...)^2}{50}+2(14.7920...)+30\right)^2$ ***(M1)***

THEN

$(\text{Area}=)\ 3050\ \mathrm{m}^2\ (3047.92...)$ ***A1***

Note: Follow through from part (d)(i), provided x is between 0 and 70. Award at most ***M1A0*** if their answer is outside the range of their $[0,\ 4713.33\ldots]$ from part (b).

[5 marks]

Total [17 marks]

4. (a) any correct Hamiltonian cycle e.g. ABCDEFA ***A1***

[1 mark]

(b) no, since not all vertices have an even degree (or equivalent) ***R1***

[1 mark]

(c) (i) 49 ***A1***

(ii) 34 ***A1***

(iii) 50 ***A1***

[3 marks]

(d) cycle is EBCDFAE

$UB = 12 + 25 + 17 + 34 + 18 + 35$ ***(M1)(A1)***

Note: Award ***M1*** for $12 + 25 + 17 + ...$ **OR** EBCD.

$= 141$ ***A1***

[3 marks]

(e) attempt to find MST for vertices A, B, C, D and E ***M1***

$12 + 14 + 17 + 27 \ (= 70)$ ***A1***

$LB = 70 + 18 + 22$ ***(M1)***

$= 110$ ***A1***

[4 marks]

(f) **EITHER**

deleting a different vertex ***A1***

might give a higher value (and hence a better lower bound). ***R1***

OR

the edges selected in part (e) do not form a cycle. ***A1***

so a higher value is possible ***R1***

[2 marks]

[14 marks]

5. (a) $\frac{1}{2}x^3+1=(x-1)^4$ ***(M1)***

$(p=)\ 2.91\ \text{cm}\ (2.91082...)$ ***A1***

[2 marks]

(b) attempt to make x (or x^2) the subject of $y=\frac{1}{2}x^3+1$ ***(M1)***

$x=\sqrt[3]{2(y-1)}$ (or $x^2=\left(2(y-1)\right)^{\frac{2}{3}}$) ***(A1)***

(upper limit $=$) $13.3(315\ldots)$ ***(A1)***

$V=\int_1^{13.3315...}\pi\left(2(y-1)\right)^{\frac{2}{3}}\ \mathrm{d}y$ ***(M1)***

Note: Award ***(M1)*** for setting up correct integral squaring their expression for x with both correct lower limit and their upper limit, and π. Condone omission of $\mathrm{d}y$.

$=197\ \text{cm}^3\ (196.946...)$ ***A1***

[5 marks]

(c) $x=y^{\frac{1}{4}}+1$ (or $x^2=\left(y^{\frac{1}{4}}+1\right)^2$) ***(A1)***

$V_2=\int_0^{13.3315...}\pi(y^{\frac{1}{4}}+1)^2\ \mathrm{d}y$ ***(M1)(A1)***

Note: Award ***(M1)*** for setting up correct integral squaring their expression for x with their upper limit, and π. Award ***(A1)*** for lower limit of 0, dependent on ***M1***. Condone omission of $\mathrm{d}y$. If a candidate found an area in part (b), do not award ***FT*** for another area calculation seen in part (c).

$=271.87668\ldots$ ***(A1)***

Note: Accept $271.038\ldots$ from use of 3sf in the upper limit.

subtracting their volumes ***(M1)***

$271.87668\ldots-196.946...$

$=74.9\ \text{cm}^3\ (74.93033\ldots)$ ***A1***

Note: Accept any answer that rounds to $75\ (\text{cm}^3)$. If a candidate found an area in part (b), do not award ***FT*** for another area calculation seen in part (c).

[6 marks]

[13 marks]

6. (a) wood layer, $W \sim \mathrm{N}(7,\ 0.3^2)$; plastic, $P \sim \mathrm{N}(3,\ 0.16^2)$

door: $X = W + P$

$\mathrm{E}(X) = 10$ (mm) ***(A1)***

$\mathrm{Var}(X) = \mathrm{Var}(W) + \mathrm{Var}(P) = 0.1156$ (mm^2) ***(M1)(A1)***

recognizing the distribution is Normal, with their mean and variance ***(M1)***

$X \sim \mathrm{N}(10,\ 0.34^2)$

$\mathrm{P}(X < 9.5) = 0.0707$ $(0.07070125...)$ ***A1***

[5 marks]

(b) $\mathrm{E}(T) = 80$ ***(A1)***

$\mathrm{Var}(T)\left(= 0.1156 \times 8\right) = 0.9248$ ***(M1)(A1)***

$T \sim \mathrm{N}(80,\ 0.9248)$

$\mathrm{P}(T > 82) = 0.0188$ $(0.0187753\ldots)$ ***A1***

[4 marks]

(c) (i) 6.93 mm $(6.93428\ldots)$ ***A1***

(ii) $(s_{n-1} =)$ 0.404 ***(A1)***

$(s_{n-1}^2 =)$ 0.163 mm^2 $(0.162928\ldots)$ ***A1***

[3 marks]

(d) $\mathrm{H}_0 : \mu_A = \mu_B$ and $\mathrm{H}_1 : \mu_A > \mu_B$ ***A1A1***

Note: Award ***A1*** for use of μ or in words "population mean", and ***A1*** for both correct equality in null hypothesis and correct inequality in alternative hypothesis. Accept an equivalent statement in words, must include mean and reference to "**population** mean" / "mean for **all** Machine B layers" for the first ***A1*** to be awarded.

use a two-sample *t*-test ***(M1)***

p-value $= 0.406975...$ ***A1***

since $0.406975... > 0.05$ **OR** p-value > 0.05 ***R1***

Do not reject H_0 (Insufficient evidence to support the employee's claim) ***A1***

Note: Accept a *p*-value of $0.415861\ldots$ from use of 3sf values from part (c). Follow through within the question for the final ***R1*** and ***A1*** for their *p*-value provided $0 \le p \le 1$. Do not award ***R0A1***.

[6 marks]

Total [18 marks]

7. (a) (i) use of chain rule ***(M1)***

$\boldsymbol{v} = -9\sin(3t)\boldsymbol{i} + 12\cos(3t)\boldsymbol{j}$ ***A1***

Note: Award ***(M1)*** for at least one correct term seen but condone omission of $\boldsymbol{i}$ or $\boldsymbol{j}$.

(ii) $|\boldsymbol{v}| = \sqrt{(-9\sin(9))^2 + (12\cos(9))^2}$ ***(M1)***

$= 11.5 \text{ m s}^{-1}$ (11.5455...) ***A1***

[4 marks]

(b) (i) $\boldsymbol{a} = -27\cos(3t)\boldsymbol{i} - 36\sin(3t)\boldsymbol{j}$ ***A1***

(ii) $\boldsymbol{a} = -9(3\cos(3t)\boldsymbol{i} - 4\sin(3t)\boldsymbol{j})$ ***M1***

$\boldsymbol{a} = -9\boldsymbol{r}$ (where $\boldsymbol{r}$ is a position vector from the origin) ***A1***

$\boldsymbol{a}$ is in opposite direction to the position vector ***R1***

hence $\boldsymbol{a}$ is always directed towards the origin ***AG***

[4 marks]

(c) relative position $\boldsymbol{d} = \boldsymbol{r}_2 - \boldsymbol{r}_1$ ***(M1)***

distance between particles $= |\boldsymbol{d}| \ (= |\boldsymbol{r}_2 - \boldsymbol{r}_1|)$ ***(M1)***

$|\boldsymbol{d}| = \sqrt{(-4\sin(4t) - 3\cos(3t))^2 + (3\cos(4t) - 4\sin(3t))^2}$ ***(A1)***

minimum value of $|\boldsymbol{d}|$ when $t = 4.71(\text{s}) \left(4.71238..., \frac{3\pi}{2}\right)$ ***(M1)A1***

[5 marks]

(d) (i) for 2nd particle, $\boldsymbol{v} = -16\cos(4t)\boldsymbol{i} - 12\sin(4t)\boldsymbol{j}$ ***(A1)***

EITHER

consider the gradient of either $\boldsymbol{v}$ ***(M1)***

$m_1 = -\frac{12\cos(3t)}{9\sin(3t)}$ and $m_2 = \frac{12\sin(4t)}{16\cos(4t)}$ ***(A1)***

attempt to solve $m_1 = m_2$ ***(M1)***

OR

vectors are parallel therefore one is a multiple of the other, $\boldsymbol{v}_2 = l\,\boldsymbol{v}_1$ ***(M1)***

$(l =) \ \frac{16\cos(4t)}{9\sin(3t)} = -\frac{\sin(4t)}{\cos(3t)}$ ***(A1)***

attempt to solve ***(M1)***

THEN

$t = 1.30 \text{ s}$ (1.30135...) ***A1***

continued…

Question 7 continued

(ii) **EITHER**

at $t = 1.30$, $\boldsymbol{v}_1 = 6.22\boldsymbol{i} - 8.68\boldsymbol{j}$ and $\boldsymbol{v}_2 = -7.57\boldsymbol{i} + 10.6\boldsymbol{j}$ ***A1***

OR

$l = -1.22$ (following second method in part (d)(i)) ***A1***

THEN

$\boldsymbol{v}_2$ is a negative multiple of $\boldsymbol{v}_1$ $(\boldsymbol{v}_2 = -1.22\boldsymbol{v}_1)$ ***R1***

the two particles are moving in the opposite direction ***AG***

[7 marks]

Total [20 marks]